Irregular Magneto-Optical Waveguides

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Abstract—**The theory of wave propagation along irregular anisotropic guides (fibers) is discussed in brief. The problem in question is considered by spectral expansion, which is constructed, in turn, with the help of the scattering operator of conical waves. The general relations are illustrated by example of a round gyrotropic waveguide of a small cross section.**

I. Introduction

ANISOTROPIC dielectric waveguides (fibers) have found wide use in optical systems (for example, in integrated optics [1]). The following paper analyzes propagation of surface modes in such structures.

II. S-Operator

First consider an open regular waveguide (Fig. 1). For large r medium parameters are assumed to coincide with free-space parameters: $\epsilon=\epsilon_v$, $\mu=\mu_v$. As has been shown in an earlier publication [2] for open waveguides with isotropic media, far from the guide axis (Fig. 1) the eigenmodes are made up (in general) of two conical waves: convergent and divergent. For $r\to\infty$ the fields of these waves are expressed in terms of Hertz functions:[1] $u_\kappa^{(\pm)}(\varphi)e^{\pm i\kappa r}/\sqrt{r}$ and $v_\kappa^{(\pm)}(\varphi)e^{\pm i\kappa r}/\sqrt{r}$ (κ is a transverse wavenumber). We can set unambiguous correspondence between these conical waves and two column matrices which are connected, in turn, with a scattering S-operator [2]

$$\hat{S}\begin{pmatrix} u_\kappa^{(-)} \\ v_\kappa^{(-)} \end{pmatrix} = \begin{pmatrix} u_\kappa^{(+)} \\ v_\kappa^{(+)} \end{pmatrix}. \tag{1}$$

In this formula convergent conical waves are indicated as the index $(-)$ and divergent ones are indicated by the superscript $(+)$. Amplitude functions of divergent waves corresponding to eigenmodes satisfy the operator equation

$$\hat{S}^{-1}\begin{pmatrix} u_{m\kappa} \\ v_{m\kappa} \end{pmatrix} = \Gamma_m(\kappa)\begin{pmatrix} u_{m\kappa} \\ v_{m\kappa} \end{pmatrix} \tag{2}$$

where $\Gamma_m(\kappa)$ is an eigenvalue of the inverse operator $\hat{S}^{-1}$ [2], m is a discrete index. Thus the eigenmodes are defined as the waves corresponding to the eigenmatrices (2). The fields of these waves are indicated as $\vec{E}_{m\kappa}$, $\vec{E}_{mp}$, etc. The eigenmode spectrum consists of continuous and discrete

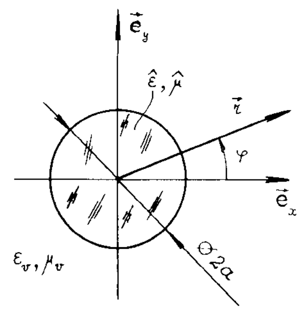

Fig. 1. Cross section of a waveguide in cylindrical coordinate (r,φ,z) system.

parts. For the modes of continuous spectrum the parameter κ assumes all real positive values [2].

Such a construction of eigenmodes can be applied for waveguides (fibers) with anisotropic (gyrotropic) media, the permittivity and permeability of which are tensors $\hat{\epsilon}$ and $\hat{\mu}$. The fields of continuous spectrum eigenmodes in such structures obey the following equations:

$$\nabla_\perp\times\vec{E}-i\omega\hat{\mu}\vec{H}=ih\vec{E}\times\vec{e}_z$$
$$\nabla_\perp\times\vec{H}+i\omega\hat{\epsilon}\vec{E}=ih\vec{H}\times\vec{e}_z \tag{3}$$

and satisfy the generalized radiation condition for $r\to\infty$

$$F+\frac{i}{\kappa}\frac{\partial F}{\partial r}=\Gamma_m(\kappa)\left[F-\frac{i}{\kappa}\frac{\partial F}{\partial r}\right]e^{-2i\kappa r}, \qquad F=\begin{pmatrix} E_z \\ H_z \end{pmatrix} \tag{4}$$

which result from (2). The subscript $\perp$ indicates transverse parts of vectors and operators; $\vec{e}_x$, $\vec{e}_y$, and $\vec{e}_z$ are unite coordinate vectors (Fig. 1). In accordance with (2),(4) the solutions of (3) far from the guide axis have the form

$$\begin{pmatrix} E_z \\ H_z \end{pmatrix} = \frac{\kappa^2}{2}\sum_{n=-\infty}^{+\infty}\begin{pmatrix} B_n \\ B_n' \end{pmatrix} \cdot\left[2J_n(\kappa r)+\left(\frac{i^{2n+1}}{\Gamma_m(\kappa)}-1\right)H_n^{(1)}(\kappa r)\right]e^{in\varphi} \tag{5}$$

where J_n, $H_n^{(1)}$ are cylindrical functions. Note, that the condition (4) can't be replaced by the inequalities $|E|,|H|<$constant, since in that case the definition of continuous spectrum eigenmodes becomes incomplete (these modes will have a denumerable degeneration). The modes of discrete spectrum are defined from the equation

$$\hat{S}^{-1}\begin{pmatrix} u_{mp} \\ v_{mp} \end{pmatrix}=0, \quad \Gamma_m(\kappa_{mp})=0, \quad \mathrm{Im}\,\kappa_{mp}>0, \qquad p=1,2,\cdots. \tag{6}$$

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[1]We shall omit the wavefactor $\exp[i(hz-\omega t)]$ (h is the propagation constant, $h^2=k^2-\kappa^2$, $k=\omega\sqrt{\epsilon_v\mu_v}$).

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These modes are ordinary surface waves [4].

If the medium is not magnetic ($\hat{\mu}\equiv\mu_v$) the equations (3) can be converted into integral form [10]

$$\vec{E}(\vec{r})=\vec{E}^{(0)}(\vec{r})+\frac{i}{4}e^{-ihz}(\nabla\nabla+k^2)e^{ihz}\cdot\int_{z=\text{const}}\hat{\chi}\vec{E}(\vec{r}')H_0^{(1)}(\kappa\rho)\,dx'dy' \quad (7)$$

$$\vec{r}=(x,y),\quad \vec{r}'=(x',y')$$

$$\rho=|\vec{r}-\vec{r}'|,\quad \hat{\chi}=\hat{\epsilon}(\vec{r}')/\epsilon_v-\hat{1}$$

where $\vec{E}^{(0)}$ is a superposition of free-space eigenmodes fields. The expression for $\vec{E}^{(0)}$ is obtained from (5) by eliminating all the terms with Hankel functions. The relation (7) is equivalent to a representation of the scattering operator in the form $\hat{S}=\hat{I}+\hat{T}_s$, where $\hat{I}$ is the unit operator. If the norm of the operator $\hat{T}_s$ is small, then the eigenmodes are constructed with the help of (7) by an iteration method (Born approximation).

By means of the S-operator we can show that the system of eigenmodes defined above is orthogonal (see the full proof in [2], [10]):

$$\int_{z=\text{const}}(\vec{E}_{m\kappa}^{(1)}\times\underset{\sim}{\vec{H}}_{l\kappa'}^{(2)}-\underset{\sim}{\vec{E}}_{l\kappa'}^{(2)}\times\vec{H}_{m\kappa}^{(1)})\vec{e}_z\,dx\,dy=D_m^{(1)}(\kappa)\,\delta_{ml}\,\delta(\kappa-\kappa')$$

$$4\pi\omega\kappa^2h\Gamma_m(\kappa)\int_0^{2\pi}(\epsilon_v u_{m\kappa}^{(1)}\underset{\sim}{u}_{l\kappa}^{(2)}-\mu_v v_{m\kappa}^{(1)}\underset{\sim}{v}_{l\kappa}^{(2)})\,d\varphi=-D_m^{(1)}(\kappa)\,\delta_{ml}. \quad (8)$$

In the formulas (8) by the underline tilde signs we indicate the eigenmodes fields of the waveguide with transposition values of tensors $\tilde{\hat{\epsilon}}$ and $\tilde{\hat{\mu}}$. In these expressions the superscripts (1) refer to the forward modes and the superscripts (2) refer to the backward ones. The orthogonality of waves with different discrete indexes m, l follows from symmetrical property of the S-operator [2], [3]. The normalizing factor $D_m^{(1)}(\kappa)$ is expressed in terms of asymptotic values of the eigenmodes fields. With the help of the orthogonal relations (8) the problems of open guides excitation by external sources may be solved [2], [10]. Note, that by means of the S-operator the problem of waveguide excitation may be considered as the diffraction problem of a superposition of the "primary" conical waves $\vec{E}^{(p)}$, $\vec{H}^{(p)}$ by the dielectric cylinder [10] ($\vec{E}^{(p)}$, $\vec{H}^{(p)}$ are the fields excited in the free space).

The relations presented above apply to the open transmission lines of an arbitrary structure. Now we assume the waveguide under consideration has the circular cross section. We also suppose that its radius a and permittivity tensor satisfy the following condition ($\hat{\mu}=\mu_v$):

$$ka\sqrt{\max|\hat{\chi}(\vec{r})|}\ll 2.4,\qquad \hat{\chi}=\hat{\epsilon}/\epsilon_v-\hat{1} \quad (9)$$

where $\hat{1}$ is unit tensor and $\max|\hat{\chi}|$ is the absolute maximum magnitude of the tensor elements. Outside the fiber the fields are sought in terms of a superposition of conical waves (5). For the condition $ka\ll 1$ the fields inside the dielectric ($r<a$) may be obtained in quasistatic approximation or in terms of power series of x and y [10]. If $ka\sim 1$, but $\max|\hat{\chi}|\ll 1$, then the fields may be calculated by the method of successive approximations. Matching the fields for $r=a\pm 0$ (Fig. 1) by conventional techniques and taking into account the condition at infinity (4) we can get a system of linear equations for the unknown coefficients $\{B_n, B_n'\}$ of the expansion (5). Setting as usual the determinant of this system equal to zero, we obtain the equation defining the eigenvalues $\Gamma_m(\kappa)$. By solving this equation and the system then we can define the structure of the eigenmodes [10]. All the calculations become simpler when taking into account the symmetrical properties of the S-operator and the orthogonality of the functions $u_{m\kappa}(\varphi), v_{m\kappa}(\varphi)$. Note that the technique of calculations used here has much in common with the technique applied in a scattering problem of quantum mechanics [5].

The eigenmodes can be also computed by (7) if $\hat{\mu}=\mu_v$. When the z axis coincides with a principal axis of the permittivity tensor $\hat{\epsilon}$ then in the first approximation this equation has the following form [10]:

$$\vec{E}(\vec{r})=\vec{E}^{(0)}(\vec{r})+\xi\int_{z=\text{const}}\hat{\chi}_\perp\vec{E}(\vec{r}')\,dS'_\perp+\frac{\nabla_\perp}{2\pi}\int_{z=\text{const}}\hat{\chi}_\perp\vec{E}(\vec{r}')\nabla_\perp\ln\frac{a}{\rho}\,dS'_\perp,$$

$$\xi=\frac{k^2}{2\pi}\ln\frac{2i}{\gamma_E\kappa a},\qquad \gamma_E=1.781\cdots,\quad dS'_\perp=dx'dy',$$

$$\rho=|\vec{r}-\vec{r}'|. \quad (10)$$

In (10) the second term is significant since the parameter κ can take an exponentionally small value and $\xi a^2\gg 1$. It follows that in general case for $ka\ll 1$ the fields of eigenmodes do not coincide with the static fields in the vicinity of the z axis. Assuming $\vec{E}^{(0)}=0$ in (10) and taking into account the conditions (6) the modes of discrete spectrum can be calculated [11].

Let us consider the eigenmodes in a circular lossless gyrotropic fiber with a radius a; for $r<a$ its permittivity tensor $\hat{\epsilon}$ is assumed to have the form ($\hat{\mu}\equiv\mu_v$)

$$\epsilon_{xx}=\epsilon_{yy}=\epsilon_1\cdot\epsilon_v$$

$$\epsilon_{xy}=-\epsilon_{yx}=i\beta\cdot\epsilon_v$$

$$\epsilon_{zz}=\epsilon_3\cdot\epsilon_v \quad (11)$$

where ϵ_1, β, and ϵ_3 are real positive constants. Other tensor components are equal to zero. Performing the calculations described above [10] for the forward symmetrical modes of the continuous spectrum $E_{0\kappa}^{(1)}$ we get the following expressions for the fields:

$$\vec{E}_\perp=\frac{h\epsilon_3}{2i\epsilon_1}\kappa^2B_0\vec{r},\quad E_z=\kappa^2B_0$$

$$\Gamma_0(\kappa)=i\exp\left[\frac{\pi i}{2}(\kappa a)^2(\epsilon_3-1)\right],\qquad \text{for } r<a \quad (12a)$$

$$E_z=\kappa^2B_0\left[J_0(\kappa r)+\frac{\pi i}{4}(\kappa a)^2(\epsilon_3-1)H_0^{(1)}(\kappa r)\right]$$

$$H_z=0,\qquad \text{for } r>a. \quad (12b)$$

For the magnetic modes $H_{0\kappa}^{(1)}$ we get

$$\vec{E}_\perp = \frac{\omega\mu_v}{2i\epsilon_1}\kappa^2 B_0'(i\beta\vec{r}-\epsilon_1\vec{e}_z\times\vec{r}), \qquad \text{for } r<a \tag{13a}$$

$$H_z \simeq \kappa^2 B_0' J_0(\kappa r), \quad E_z=0, \quad \Gamma_0(\kappa)=i, \qquad \text{for } r>a. \tag{13b}$$

The fields of the nonsymmetrical modes $HE_{1\kappa}^{(\pm 1)}$ are equal to

$$\vec{E}_\perp = \frac{i\kappa B_1(k^2+h^2)(\vec{e}_x \pm i\vec{e}_y)}{h(\epsilon_1 \mp \beta - 1)(ka)^2 \ln\left(\kappa/\kappa_{11}^{(\pm 1)}\right)}$$

$$\Gamma_1(\kappa) = \frac{\ln\left(\kappa/\kappa_{11}^{(\pm 1)}\right)}{i\ln\left(-\kappa/\kappa_{11}^{(\pm 1)}\right)}, \qquad \text{for } r<a \tag{14a}$$

$$E_z = \frac{\kappa^2}{2}B_1 e^{\pm i\varphi}\left[2J_1(\kappa r) - \left(\frac{i}{\Gamma_1(\kappa)}+1\right)H_1^{(1)}(\kappa r)\right]$$

$$H_z = \mp\frac{\omega\epsilon_v}{h}E_z, \qquad r>a$$

$$\kappa_{11}^{(\pm 1)}a \simeq \frac{2i}{\gamma_E}\exp\left[\frac{\epsilon_3+1}{8} - \frac{\epsilon_1 \mp \beta + 1}{(ka)^2(\epsilon_1 \mp \beta - 1)}\right]. \tag{14b}$$

The subscript indexes $(+1)$ and (-1) refer to the forward modes with the right and left polarizations, respectively. In the formulas (14) $\kappa_{11}^{(+1)}$ and $\kappa_{11}^{(-1)}$ are the transverse wavenumbers of the surface modes $HE_{11}^{(+1)}$ and $HE_{11}^{(-1)}$; they are roots of the equations $\Gamma_1(\kappa)=0$. Other modes are calculated in a similar way. For example, the fields of the modes $HE_{2\kappa}^{(\pm 1)}$ are equal to

$$E_x = \frac{i\kappa^2}{2h}\cdot\frac{B_2 r}{\epsilon_{ef}+1}(k^2+h^2)e^{\pm i\varphi}$$

$$E_y = \pm iE_x, \qquad \text{for } r<a \tag{15a}$$

$$E_z = \kappa^2 B_2 J_2(\kappa r)e^{\pm 2i\varphi}$$

$$H_z = \mp\frac{\omega\epsilon_v}{h}E_z, \qquad \text{for } r \gg a \tag{15b}$$

where $\epsilon_{ef} = \epsilon_1 \mp \beta$ are "effective" permittivity values.

III. Irregular Waveguides

With the help of the mode systems constructed above the problem of surface mode transformation in irregular fibers may be considered. To solve the problem in question in each cross section of irregular line the fields are expressed in terms of the local eigenmodes of continuous and discrete spectrum [2] which correspond to the local waveguide structure [6]–[8]

$$\vec{E} = \sum_{\alpha=1,2}\left[\sum_{m,p} C_{mp}^{(\alpha)}(z)\vec{E}_{mp}^{(\alpha)} + \sum_m \int_0^\infty C_{m\kappa}^{(\alpha)}(z)\vec{E}_{m\kappa}^{(\alpha)}\,d\kappa\right]. \tag{16}$$

By substituting the expansion (16) in Maxwell's equations we obtain [6] the infinite system of the coupled mode equations for the amplitudes of the local eigenmodes $C_{mp}^{(\alpha)}(z)$ and $C_{m\kappa}^{(\alpha)}(z)$. For instance, the equations for the amplitudes of forward radiation modes $(0<\kappa<k)$ have the following form (in the first approximation):

$$\frac{dC_{m\kappa}^{(1)}}{dz} - ih_\kappa C_{m\kappa}^{(1)} = Sct_{m\kappa,11}^{(1,1)}(z)C_{11}^{(1)} + \cdots \tag{17}$$

$$Sct_{m\kappa,11}^{(1,1)}(z) = \omega\left[(h_{11}-h_\kappa)D_m^{(1)}(\kappa)\right]^{-1} \cdot \int_{z=\text{const}} \vec{E}_{m\kappa}^{(2)}\left(\frac{\partial\hat{\epsilon}}{\partial z}\vec{E}_{11}^{(1)}\right)dx\,dy \tag{18}$$

where $C_{11}^{(1)}(z)$ is an amplitude of the incident surface mode ($HE_{11}^{(+1)}$ or $HE_{11}^{(-1)}$). These equations describe the mode coupling, including the coupling of surface and radiation modes. According to the type of guide imperfections the coupling coefficients $Sct_{m\kappa,11}^{(1,1)}$ can be converted to a form which is more convenient for calculations [6], [7]. For small discontinuities or slowly changing irregularities the system (17) may be solved by the method of successive approximations [6]–[9] (first assuming $|C_{11}^{(1)}| \simeq 1$, $C_{11}^{(2)} = C_{m\kappa}^{(\alpha)} = 0$). Then substituting the computed amplitudes in expansion (16) and using the method of stationary phase we can estimate the far fields and the power losses [2], [6], [10]. It should be noted that far from the guide axis the fields have the form of a spherical wave; over area $r \sim a$ the fields are a superposition of surface modes and space waves [10]. The amplitudes of the latter decrease as

$$\frac{\exp(ik|z|)}{|z|(\ln|z|)^2}, \qquad \text{for } z \to \pm\infty$$

(see the details in [10]).

Let us apply the method described above to the problem of surface mode propagation in an irregular magneto-optical fiber. We assume that this guide is circular and its parameters satisfy the relations (9) and (11). The equations (17) are solved in the same way as in the papers [6], [9] dealing with a waveguide with an isotropic dielectric. Therefore we omit the intermediate calculations and give here only the final results. An incident surface mode $HE_{11}^{(+1)}$ (or $HE_{11}^{(-1)}$) is assumed to propagate from $z=-\infty$ to the irregular fiber section. In what follows the upper signs correspond to the case, when the incident wave has right circular polarization, and the lower signs refer to the case, when the wave has left circular polarization.

First consider the fiber with a small step of the diameter. For small values of polar angle ϑ ($\tan\vartheta = z/r$) the radiation pattern of scattering spherical wave has the following form:

$$f(\vartheta) \simeq f_a\left[(\vartheta^2+\vartheta_{11}^2)^2(\pi^2+4(\ln|\vartheta/\vartheta_{11}|)^2)\right]^{-1}, \qquad \vartheta_{11} = \left|\frac{\kappa_{11}^{(\pm 1)}}{k}\right| \tag{19}$$

(f_a is a constant). By integrating the function (19) we

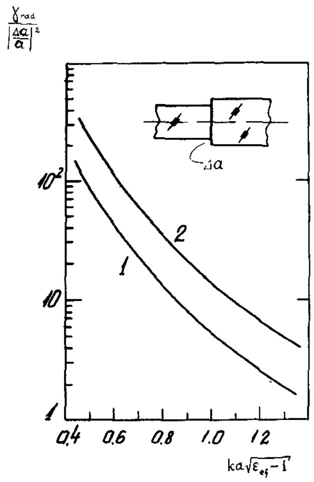

Fig. 2. Radiation losses as a function of dimensionless frequency $ka\sqrt{\epsilon_{ef}-1}$ for a step in guide diameter ($\epsilon_{ef}=\epsilon_1\mp\beta$).

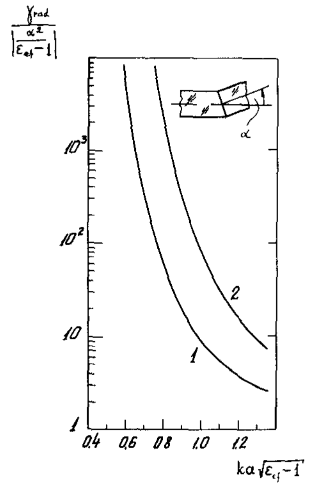

Fig. 3. Radiation losses for a small corner for $\epsilon_{ef}=1.01$ (curve 1) and for $\epsilon_{ef}=2.25$ (curve 2).

obtain the expression for the relative power losses [9], [10]

$$\gamma_{rad}=\frac{4}{3}\left[\frac{\epsilon_1\mp\beta+1}{(ka)^2(\epsilon_1\mp\beta-1)}\right]^2\cdot\left|\frac{\Delta a}{a}\right|^2. \quad (20)$$

Fig. 2 presents the power radiation losses γ_{rad} as a function of dimensionless frequency $ka\sqrt{\epsilon_{ef}-1}$ for two values of effective permittivity ϵ_{ef}. The curves are constructed for $\epsilon_{ef}=1.01$ (line 1) and for $\epsilon_{ef}=2.25$ (line 2). For simplicity when computing the curves in Fig. 2 we assume that $\epsilon_3=\epsilon_{ef}$. As seen from (20) for $\epsilon_1-\beta\simeq 1$ the mode $HE_{1\kappa}^{(+1)}$ with right circular polarization is unstable in a guide with a changing diameter. It follows that optical fibers with such parameters have only one stable polarization of propagating modes (the left polarization). For these irregularities the reflection coefficient of surface modes is exponentionally small [6].

For an axis offset in the plane (x,z) at a value Δx and for a corner with a small angle α the relative power losses are equal, respectively, to

$$\gamma_{rad}^{(offset)}=\frac{\epsilon_1\mp\beta+1}{\epsilon_1\mp\beta-1}\vartheta_{11}^2\left|\frac{\Delta x}{a}\right|^2,\quad \gamma_{rad}^{(corner)}=\frac{1}{3}\left|\frac{\alpha}{\vartheta_{11}}\right|^2 \quad (21)$$

($|\Delta x|\ll a$, $\vartheta_{11}\ll 1$). Fig. 3 shows the radiation loss coefficient versus the dimensionless frequency for the corner ($\epsilon_3=\epsilon_{ef}$). In this case for $\vartheta\ll 1$ the power pattern $f(\vartheta)$ has the form

$$f(\vartheta)\simeq f_\alpha\vartheta^2(\vartheta^2+\vartheta_{11}^2)^{-4}$$
$$\vartheta_{11}=|\kappa_{11}^{(\pm1)}/k|$$
$$f_\alpha=\text{constant}. \quad (22)$$

This function is presented in Fig. 4.

The given results can be generalized for slowly changing transitions of the fiber. In this case the radiation losses substantially depend on analytic properties of irregularity geometry; for smooth transitions the power losses are exponentionally small in magnitude. For example, if the

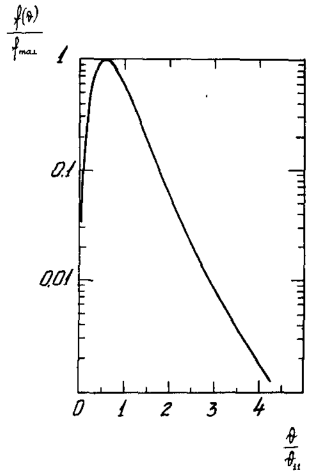

Fig. 4. The power scattering pattern for a waveguide corner (f_{max} is a maximum of the function).

fiber axis is bent along the line

$$x_{axis}=\frac{d}{(1+z^2/l^2)}$$

then for $d\ll l\vartheta_{11}{}^2$ and $kl\vartheta_{11}^2\gg 1$ the relative power losses are equal to

$$\gamma_{rad}=\frac{\pi^2}{32}(kd)^2\vartheta_{11}^2\exp(-kl\vartheta_{11}^2). \quad (23)$$

Note that for the irregularities caused by diameter change the radiation modes $HE_{1\kappa}$ and $EH_{1\kappa}$ are excited [6], [9]; in other cases considered above the modes $E_{0\kappa}$, $H_{0\kappa}$, $HE_{2\kappa}$, and $EH_{2\kappa}$ are excited.

The description of the mode transformation is not unique. For some types of irregularities the ideal mode expansion

[2]This inequality is equivalent to

$$\left|\frac{d}{dz}x_{axis}\right|\ll\vartheta_{11}.$$

and perturbation method are particularly well suited for calculations [2]. Using such a technique for a small gap in the waveguide, the radiation losses are [9]

$$\gamma_{\mathrm{rad}} = \frac{8}{3}\cdot\frac{|\kappa_{11}^{(\pm 1)}\Delta l|^2}{(\epsilon_{ef}+1)^2}, \qquad \Delta l \ll a \tag{24}$$

where Δl is a length of the gap.

IV. Conclusion

We have calculated the mode transformation in an irregular gyrotropic fiber. In a similar way the surface mode propagation can be calculated for a waveguide, whose permittivity tensor is real and diagonal [10]. Under the condition (9) the formulas for the radiation losses have a similar form to the ones given above. Note that in general case this fiber has no pure symmetrical modes.

The considered above method of the S-operator can be applied in solving quite a number of problems [10]. The calculations may be simplified if we take into account characteristic properties of the fiber. In particular, when the permittivity tensor $\hat{\epsilon}$ of the guide is real, the S-operator is unitary. It follows that the radiation losses, which are caused by the excitation of different modes (with different discrete indexes), are added independently. Also, in this case the mode orthogonality conditions (8) may be simplified by using complex conjugate functions. Unitary conditions on the S-operator impose specified restrictions on the Fourier coefficients of the field expansions (5) at infinity. For example, in a waveguide with arbitrary form of a cross section, the permittivity tensor of which is real and diagonal, for the radiation modes $HE_{1\kappa}$ the coefficients of the expansion (5) satisfy the relations: $hB_1' = \pm\omega\epsilon_v B_1$, $B_n = B_n' = 0$, $n > 1$, for $k \to 0$.

The properties of the S-operator used here have much in common with the properties of the S-matrix applied in quantum mechanics [5], [12], but there exist many differences. In particular, analytic properties of the S-operator are more complicated than those of the matrix, since in the complex plane of the parameter κ the functions $u_{m\kappa}$, $v_{m\kappa}$ necessarily have the branch points (for $h=0$).

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